3rd International Workshop on
Dark Matter, Dark Energy and
Matter-Antimatter Asymmetry

Cosmological perturbations in modified teleparallel theories

Chao-Qiang Geng & YPW, arXiv 1212.6214

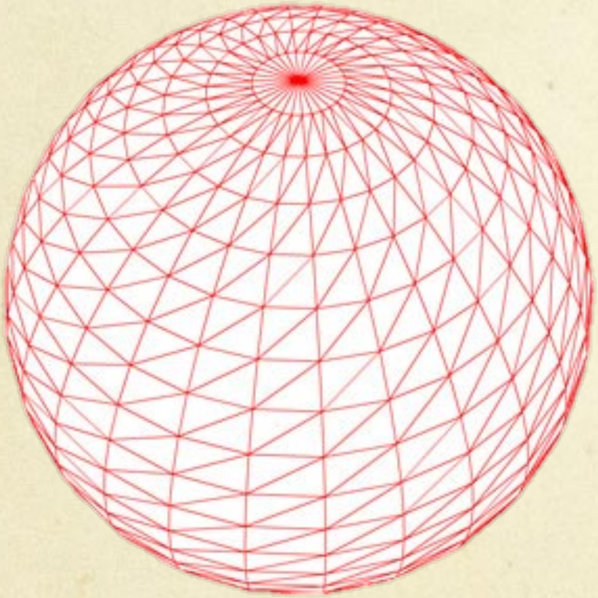
Yi-Peng Wu
National Tsing Hua University
December 30, 2012 @ NTU

Teleparallel gravity

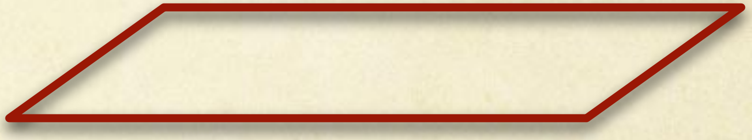
Teleparallel gravity

$$g_{\mu\nu} \neq \eta_{\mu\nu}$$

GR



Teleparallelim



GR

- metric
- Palatini

- GR
  - metric
  - Palatini

- teleparallelism
  - A Hehl et al (1995)
  - B Hayashi & Shirafuji (1979)

teleparallelism

$$\text{A}: \quad \lambda^{AB\mu\nu}\mathcal{R}_{AB\mu\nu}$$

$$\text{B}: \quad \Gamma^{\lambda}_{\mu\nu} = e^{\lambda}_{A}\partial_{\nu}e^{A}_{\mu}$$

$$\left.\begin{matrix} \text{A} \\ \text{B} \end{matrix}\right\} \mathcal{R} = 0$$

teleparallelism

$$\text{A}: \quad \lambda^{AB\mu\nu}\mathcal{R}_{AB\mu\nu}$$

$$\text{B}: \quad \Gamma^{\lambda}_{\mu\nu} = e^{\lambda}_{A}\partial_{\nu}e^{A}_{\mu}$$

$$\Bigg\} \ \mathcal{R} = 0$$

$$\Gamma^{\lambda}_{\mu\nu} = e^{\lambda}_{A}\partial_{\nu}e^{A}_{\mu}$$

- $\nabla_{e_A} e_B = 0$
- $g_{\mu\nu} = \eta_{AB}e^{A}{}_{\mu}e^{B}{}_{\nu}$
- $T^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\nu\mu} - \Gamma^{\lambda}{}_{\mu\nu}$

$$\Gamma^{\lambda}_{\mu\nu} = e^{\lambda}_{A}\partial_{\nu}e^{A}_{\mu}$$

$$e^{A}{}_{\mu}$$

➔ $e^{\bar{A}}{}_{\mu} = \Lambda^{\bar{A}}{}_{B}e^{B}{}_{\mu}$

$$g_{\mu\nu} \to g_{\bar{\mu}\bar{\nu}}$$

$\eta_{AB} = \eta_{CD}\Lambda^{C}{}_{A}\Lambda^{D}{}_{B}$

$$\Gamma^{\lambda}_{\mu\nu} \to \Gamma^{\bar{\lambda}}_{\bar{\mu}\bar{\nu}} + \delta^{\lambda}_{\mu}\Lambda\partial_{\nu}\Lambda$$

$$\Gamma^{\lambda}_{\mu\nu} = e^{\lambda}_{A}\partial_{\nu}e^{A}_{\mu}$$

$$e^{A}{}_{\mu}$$

➔ $e^{\bar{A}}{}_{\mu} \neq \Lambda^{\bar{A}}{}_{B}e^{B}{}_{\mu}$

➔ 16 degrees of freedom

$$g_{\mu\nu} \rightarrow g_{\bar{\mu}\bar{\nu}}$$

$$\eta_{AB} = \eta_{CD}\Lambda^{C}{}_{A}\Lambda^{D}{}_{B}$$

$$\Gamma^{\lambda}_{\mu\nu} \rightarrow \Gamma^{\bar{\lambda}}_{\bar{\mu}\bar{\nu}} + \delta^{\lambda}_{\mu}\Lambda\partial_{\nu}\Lambda$$

$$e^{A}_{\mu}$$

➔ 16 degrees of freedom

➔ $e^A_\mu(x) = \hat{e}^A_\mu(x) + æ^A_\mu(x)$

purely perturbations

$$g_{\mu\nu}(x) = \eta_{AB}\, e^A_\mu(x)\, e^B_\nu(x) = \eta_{AB}\, \hat{e}^A_\mu(x)\, \hat{e}^B_\nu(x)$$

FRW Universe: $e^A_\mu = \mathrm{diag}(1, a, a, a)$

$$g_{\mu\nu} = \mathrm{diag}(1, -a^2, -a^2, -a^2)$$

Vierbein perturbation

YPW & Geng, JHEP **11** (2012) 142

$$e^A_{\ \mu}$$

$$
\begin{aligned}
e^0_\mu &= \delta^0_\mu(1+\psi) + a\delta^i_\mu\partial_i(F+\alpha) + a\delta^i_\mu(G_i+\alpha_i) \\
e^a_\mu &= a\delta^a_\mu(1-\varphi) + a\delta^i_\mu(\partial_i\partial^a B + \partial^a C_i + h^a{}_i) \\
&\quad + a\delta^i_\mu B^a{}_i + \delta^0_\mu(\partial^a\alpha + \alpha^a)
\end{aligned}
$$

$$g_{\mu\nu}$$

$$
\begin{aligned}
g_{00} &= 1+2\psi \\
g_{i0} &= a(\partial_i F + G_i) \\
g_{ij} &= -a^2[(1-2\varphi)\delta_{ij} + h_{ij} + \partial_i\partial_j B + \partial_j C_i + \partial_i C_j]
\end{aligned}
$$

Vierbein perturbation

YPW & Geng, JHEP **11** (2012) 142

$$e^A_{\ \mu}$$

$$\begin{aligned} e^0_\mu &= \delta^0_\mu(1+\psi) + a\delta^i_\mu\partial_i(F+\alpha) + a\delta^i_\mu(G_i+\alpha_i) \\ e^a_\mu &= a\delta^a_\mu(1-\varphi) + a\delta^i_\mu(\partial_i\partial^a B + \partial^a C_i + h^a{}_i) \\ &\quad + a\delta^i_\mu B^a{}_i + \delta^0_\mu(\partial^a\alpha + \alpha^a) \end{aligned}$$

new members : α , α_i and $B^a{}_i$

1 + 2 + 3 = 6

$$\partial_i\alpha^i = 0\,;\, B_{ij} + B_{ji} = 0$$

α : Scalar mode

α_i : Decaying mode

${B^a}_i$: pseudo-scalar and pseudo-vector mode

$$B_{ij} = \epsilon_{ijk}(\partial^k \tilde{\sigma} + \tilde{V}^k)$$

➔ Do not involve in linear perturbations

Izumi & Ong, arXiv 1212.5774

Teleparallel equivalence of general relativity (TEGR)

$$\Gamma^{\lambda}_{\mu\nu} = e^{\lambda}_{A}\partial_{\nu}e^{A}_{\mu} \qquad G_{\mu\nu} = \kappa^2\Theta_{\mu\nu}$$

$$T = \frac{1}{4}T_{\rho}{}^{\mu\nu}T^{\rho}{}_{\mu\nu} - \frac{1}{2}T^{\mu\nu}{}_{\rho}T^{\rho}{}_{\mu\nu} - T^{\rho}{}_{\rho\mu}T^{\nu}{}_{\nu}{}^{\mu}$$

$$\Updownarrow$$

$$T = R + 2\bar{\nabla}^{\nu}T^{\mu}{}_{\mu\nu}$$

f(T) gravity

$$S = \frac{1}{2\kappa^2} \int d^4x\, e\, [T + f(T) + \mathcal{L}_m]$$

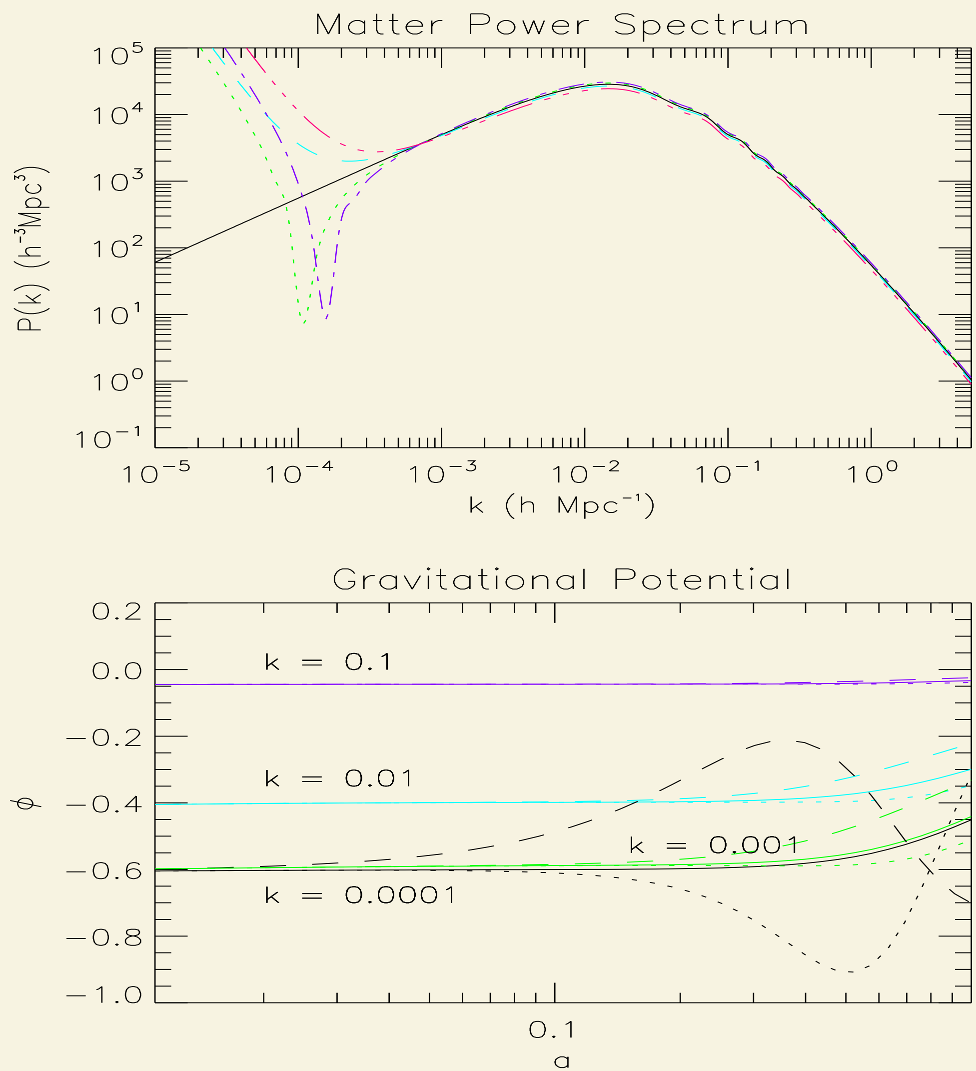
16 field equations:

$$(1 + f_T)\, G_\mu{}^\nu - \frac{1}{2}\delta_\mu{}^\nu (f - T f_T) + 2S_\mu{}^{\lambda\nu}\, (\partial_\lambda f_T) = \kappa^2 \Theta_\mu{}^\nu$$

$$\boxed{\left(g^{\mu\alpha} S_\mu{}^{\lambda\beta} - g^{\nu\beta} S_\nu{}^{\lambda\alpha}\right) \partial_\lambda f_T = 0}$$

f(T) gravity

Li, Sotiriou & Barrow PRD **83** (2011) 104017



sub-horizon scales:

$$\alpha \simeq 0$$

$$\varphi = \psi$$

super-horizon scales:

$$\varphi = \psi_s - 12\frac{\dot{H} f_{TT}}{1+f_T}\alpha_m$$

$$\alpha_m \equiv aH\alpha$$

YPW & Geng, JHEP **11** (2012) 142

Teleparallel dark energy

Geng, Lee, Saridakis & YPW, PLB **704** (2011) 384

$$S = \int d^4xe\left[\frac{T}{2\kappa^2} + \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi + \xi T\phi^2\right) - V(\phi) + \mathcal{L}_m\right]$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) - 3\xi H^2\phi^2$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) + 4\xi H\phi\dot{\phi} + \xi\left(3H^2 + 2\dot{H}\right)\phi^2$$

Teleparallel dark energy

Geng, Lee, Saridakis & YPW, PLB **704** (2011) 384

$$S = \int d^4 x e \left[\frac{T}{2\kappa^2} + \frac{1}{2}\left(\partial_\mu \phi \partial^\mu \phi + \underbrace{\xi T \phi^2} \right) - \underbrace{V(\phi)} + \mathcal{L}_m \right]$$

★

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) - 3\xi H^2 \phi^2$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) + 4\xi H \phi \dot{\phi} + \xi \left(3H^2 + 2\dot{H} \right) \phi^2$$

★ : Non-minimal gravity-field tracker (NGFT)

Gu, Geng & Lee, PLB **718** (2013) 722

Teleparallel dark energy

$$\left(\frac{1}{\kappa^2}+\xi\phi^2\right)G^{\nu}_{A}-e^{\nu}_{A}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\right]+e^{\mu}_{A}\partial^\nu\phi\partial_\mu\phi$$
$$+4\xi S_A{}^{\lambda\nu}\phi\left(\partial_\lambda\phi\right)=\Theta_A{}^{\nu}$$

$$G_{\mu\nu}=e^{A}_{\mu}G_{A\nu}$$

16 field equations:

$$4\xi\phi\left(g^{\mu\alpha}S_\mu{}^{\lambda\beta}-g^{\nu\beta}S_\nu{}^{\lambda\alpha}\right)\partial_\lambda\phi=0$$

Main results:

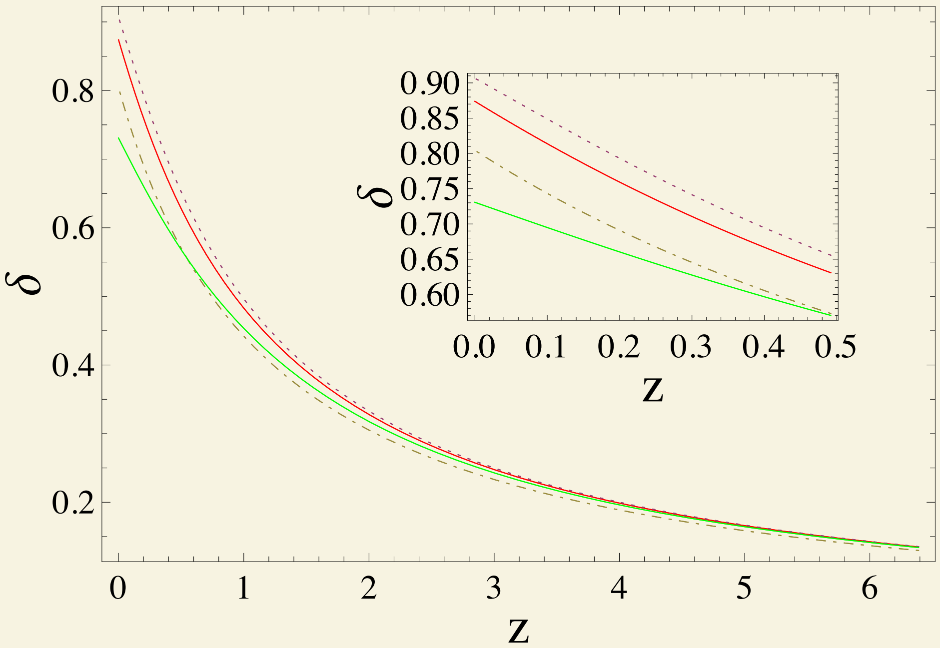
Geng & YPW, arXiv 1212.6214

$$\psi = \varphi - \frac{2\kappa^2\xi\phi\dot{\phi}}{H(1+\kappa^2\xi\phi^2)}\alpha_m \qquad \alpha_m \equiv aH\alpha$$

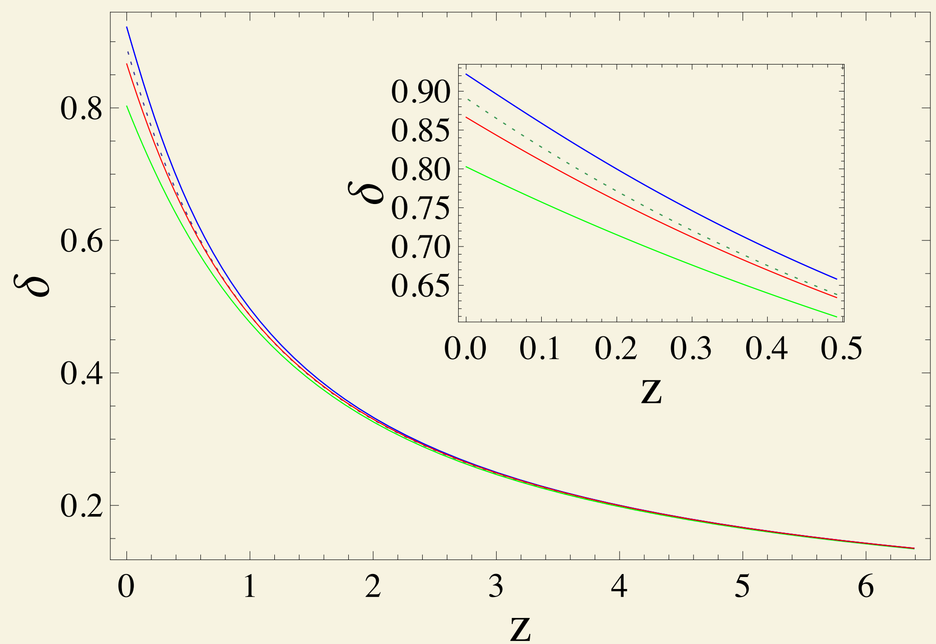
$$\varphi = -\frac{H}{\dot{\phi}}\delta\phi \qquad \delta\phi = 4\xi\phi\alpha_m$$

$$G_{\text{eff}} = \left(1 + \frac{\epsilon}{1+\kappa^2\xi\phi^2}\right)\frac{G}{1+\kappa^2\xi\phi^2} \qquad \epsilon = \kappa^2\dot{\phi}^2/2H^2$$

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}}\rho_m\delta = 0$$



0.8
0.6
0.4
0.2
δ
0 1 2 3 4 5 6
z
0.90
0.85
0.80
0.75
0.70
0.65
0.60
δ
0.0 0.1 0.2 0.3 0.4 0.5
z



0.8
0.6
0.4
0.2
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0 1 2 3 4 5 6
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0.90
0.85
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0.70
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z

My messages are:

- The propagating degrees of freedom do not increase in teleparallel dark energy models, despite variables in the perturbed vierbein field is greater than that of the metric perturbations.
- The density perturbation growth shows that gravitational interactions in potential-driven (quintessence-like) expansion are less stronger than those in NGFT-driven cosmic accelerations.

$$e^A_{\ \mu}$$

➔ $$e^A_\mu(x) = \hat{e}^A_\mu(x) + æ^A_\mu(x)$$

➔ $$g_{\mu\nu}(x) = \eta_{AB}\, e^A_\mu(x)\, e^B_\nu(x) = \eta_{AB}\, \hat{e}^A_\mu(x)\, \hat{e}^B_\nu(x)$$

$$\hat{e}^A_{\ \mu}$$

$$\begin{aligned} \hat{e}^0_\mu &= \delta^0_\mu(1+\psi) + a(G_i + \partial_i F)\delta^i_\mu \\ \hat{e}^a_\mu &= a\delta^a_\mu(1-\varphi) + a(h^a_i + \partial_i\partial^a B + \partial_i C^a + \partial^a C_i)\delta^i_\mu \end{aligned}$$

$$g_{\mu\nu}$$

$$\begin{aligned} g_{00} &= 1 + 2\psi \\ g_{i0} &= a(\partial_i F + G_i) \\ g_{ij} &= -a^2[(1-2\varphi)\delta_{ij} + h_{ij} + \partial_i\partial_j B + \partial_j C_i + \partial_i C_j] \end{aligned}$$

$$e^{A}_{\ \mu}$$

➔ $e_\mu^A(x) = \hat{e}_\mu^A(x) + æ_\mu^A(x)$

➔ $g_{\mu\nu}(x) = \eta_{AB}\, e_\mu^A(x)\, e_\nu^B(x) = \eta_{AB}\, \hat{e}_\mu^A(x)\, \hat{e}_\nu^B(x)$

$$æ_\mu^0 = (æ, æ_i)\ ;\ æ_\mu^a = (A^a, B_i^a)$$

$æ^{A}_{\ \mu}$

$$æ = 0\ ,\ æ_i = aA_i\ ,\ \text{and } B_{ij} + B_{ji} = 0$$